The 27th Balkan Mathematical Olympiad
Chisinau, Republic of Moldova
May 2-8, 2010

Grading scheme

Each problem is worth 10 points.

Problem 1

1) For proving, that initial inequality is equivalent to the inequality: ……………….…………...3 points

$$\frac{a(b-c)}{c(a+b)}+\frac{b(c-a)}{a(b+c)}+\frac{c(a-b)}{b(c+a)}\geq 0$$

In particular :

- for substitution of a,b,c by positive x,y,z that gives $xyz=1$ ……....1point
- for using of this substitution and obtaining the above inequality ..…..2points

2) For proving, that inequality from 1) is equivalent to the inequality

$$\frac{b(c+a)}{c(a+b)}+\frac{c(a+b)}{a(b+c)}+\frac{a(b+c)}{b(c+a)}\geq 3$$ ……………………..……………………...4points

3) For proving of inequality from 2) ………………………………..…………………….……3points

Grading scheme

Each problem is worth 10 points.

Problem 2

1) It is mentioned that quadrilateral $ACBH_1$ is cyclic ……………………….............1point

2) For proving that point $N = QC_1 \cap BH_1$ is the midpoint of BH_1 ………………...2points
   - For the assertion that point $N = QC_1 \cap BH_1$ is the midpoint of BH_1 …...1point
   - For proving that point $N = QC_1 \cap BH_1$ is the midpoint of BH_1 ………..1point

3) For proving that $PMRN$ is cyclic ………………...…… …………..……….…..3points
   - For the assertion that $PMRN$ is cyclic …………… ……….…..………1point
   - For proving that $PMRN$ is cyclic ………… ……………………......…..2points

4) For proving that $PMRN$ is cyclic deltoid ……………..…… ……………...…..2points
   - For the assertion that $PMRN$ is cyclic deltoid …...…… ………………....1point
   - For proving that $PMRN$ is cyclic deltoid ………………………...…......1point

5) For proving that points N and M_1 coincide ……………….………………….....2points
   - For the assertion that points N and M_1 coincide ……………..……….....1point
   - For proving that points N and M_1 coincide ….…….......……………....1point

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Note.

For non proved assertions totally not more than 2 points.

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Problem 3

I. The least width of a strip covering the triangle is equal to the length of the shortest altitude **5p**. In particular:

Ia) At least one of the perpendicular lines through the vertices to the borders meets the opposite side.1p

Ib) The altitude corresponding to that vertex is of length at most w: ..………………………………..3p

Ic)The least width of a strip covering the triangle is equal to the length of the shortest altitude……...1p

II a) Choosing the points A, B at maximal distance; the altitude from any C is the shortest one:……...3p

II b) Construction ……………………………………………………………………………….2p

Grading scheme

Each problem is worth 10 points.

Problem 4

1) A counting formula for $f(n)$ is found……………………………………………….<u>2 points</u>

2) It is proved that the equality $f(n+p) = f(n)$ implies:………………….......................<u>5 points</u>

- $n \vdots p$ ………………………………………………………………….……………..2 points
- $\varphi(k) = \dfrac{2(k+1)}{p-1} - 1$ and $\varphi(k+1) = \dfrac{2(k+1)}{p-1} + 1$ ……......................................3 points

3) It is proved that each of the following conditions leads to a contradiction: ……………..<u>3 points</u>

- $\varphi(k) \vdots 4$ and $\varphi(k+1) \vdots 4$ ……………………………………..……………...1 point
- $\varphi(k) \not\vdots\, 4$ ………………………………...……………………………..1 point
- $\varphi(k+1) \not\vdots\, 4$ ……………………………………………...………………..1 point